

Second Year of Preparatory Cycle

Exercise Sheet No. 1 (part 1) of Fluid Mechanics

Fluid Statics

General data (unless stated otherwise): $g = 9.81 \text{ m.s}^{-2}$; $\rho_{\text{water}} = 1000 \text{ kg.m}^{-3}$; $\rho_{\text{mercury}} = 13\,600 \text{ kg.m}^{-3}$; $P_{\text{atm}} = 101325 \text{ Pa}$; free surfaces are at atmospheric pressure; notation follows the lecture (R resultant force, S area, G centroid, C centre of pressure, S_i specific gravity); pressures are gage pressures unless "absolute" is specified.

Part I - Fundamentals of hydrostatics**Exercise 1 - Fluid properties and pressure scales**

A storage tank contains 2.5 m^3 of oil of specific gravity $S_i = 0.85$.

1. Calculate the mass of the oil, its weight, its specific weight (N/m^3) and its specific Volume.
2. At a point inside the oil the absolute pressure is 180 kPa . Determine the gage pressure at this point and express it as an equivalent height of water and as a height of mercury (in mm).
3. A vacuum gage connected to a closed vessel reads -35 kPa . What is the absolute pressure in the vessel?
4. Water has an isothermal compressibility $\chi_T \approx 5 \times 10^{-10} \text{ Pa}^{-1}$. Estimate the relative change in density $\Delta\rho/\rho$ when the pressure increases by 400 bar (about 4000 m of water). Is the incompressible approximation justified?

Exercise 2 - Fundamental law of hydrostatics and layered liquids

1. Consider a vertical cylindrical fluid element of cross-section dS and height dz , at rest in a fluid of density ρ . Apply the equilibrium of forces to show that $dP/dz = -\rho g$ (z pointing upwards). Deduce $P(z)$ for an incompressible fluid and state the conditions under which the result holds.
2. A closed tank contains three immiscible liquids, from top to bottom: oil ($\rho = 800 \text{ kg.m}^{-3}$, thickness 1.5 m), water (thickness 2.0 m) and glycerin ($\rho = 1260 \text{ kg.m}^{-3}$, thickness 1.0 m). The air above the oil is at a gage pressure of 30 kPa . Calculate the gage pressure at each interface and at the bottom of the tank, and sketch the graph $P(z)$.
3. A piezometer tube, open to the atmosphere, is connected to the tank at the water-glycerin interface. To what height above this interface does the water rise in the tube?

Exercise 3 - Pascal's law and hydraulic lift

A hydraulic lift is filled with oil of density $\rho = 850 \text{ kg}\cdot\text{m}^{-3}$. The large piston B, of area $S_B = 0.05 \text{ m}^2$, carries a platform and a car of total mass 1500 kg. The small piston A has an area $S_A = 10 \text{ cm}^2$ and is located $h = 0.8 \text{ m}$ above piston B.

1. Neglecting the height difference h , calculate the pressure under piston B, the mechanical advantage S_B/S_A and the force F_A needed to hold the car.
2. Taking h into account, calculate the exact force F_A .
3. To raise the car by 2 cm, how far must piston A move? Compare the work done on piston A with the work received by the car, and interpret the difference.